# PLASTIC DEFORMATION AND IRREVERSIBLE CHANGES IN A SOLID BODY IN THE PRESENCE OF LOCAL MELTING. POINT SOURCE OF HEAT

(PLASTICHESKAIA DEFORMATSIIA I NEOBRATIMYE IZMENENIIA V TVERDOM TELE PRI LOKAL'NOM PLAVLENII. TOCHECHNYI ISTOCHNIK TEPLA)

*PMM Vol.24, No.5, 1960, pp. 843-851*

A.M. KOSEVICH and L.V. TANATAROV
(Khar'kov)

*(Received 9 April 1960)*

This paper deals with plastic deformation caused by the difference of specific volumes of solid and fluid phases of matter during local melting in a solid body. It is shown that, in the process of hardening of the region of melting, large (in absolute value) negative pressures may arise in the fluid, and that they can cause discontinuity of the fluid. The latter phenomenon causes the existence of cavities in the body after full hardening.

Local melting is the melting in a small region of a solid body caused by generation of some quantity of heat in a small volume during a short period of time.

In this paper, the assumption is made that the heat is instantaneously generated at a point in the interior of an isotropic solid body. Immediately after the heat is generated, the matter in the infinitesimal volume around the origin of the coordinate system becomes melted. The spherical boundary between the fluid and the solid body of the radius $r_0$ (implied by the symmetry of the problem) moves in time in the following way: initially, $r_0$ increases in time (melting caused by temperature increment at the origin of the coordinate system), reaches its maximum value $r_m$ and then decreases (hardening caused by flow of the heat out of the zone of melting) and approaches zero.

It is assumed that the fluid phase of the matter has the specific volume larger than the specific volume of the solid phase, and that the relative increment of the dimensions $\epsilon_0$ during melting exceeds the deformation at the yield limit of the material.

Consequently, the formation of a region with a considerably "excessive" specific volume results in the creation of a zone of plastic deformation of the solid body around this region. The character of deformation during melting, when the region of the fluid phase expands (loading in a solid body), is essentially different from the character of deformation during hardening, when the region of the fluid phase decreases (unloading in a solid body).

Because of the irreversibility of the process of deformation, some permanent changes of the material occur after re-hardening of the region of melting; they will be discussed later.

In the analysis of the described phenomenon, the relaxation processes, which may arise in a solid body in high temperatures, will be neglected. This is allowable if the period of melting and hardening of the material in the region of the radius $r_*$ is smaller than the characteristic relaxation time. In addition thermal stresses will not be taken into account, as they are negligible if $a\Delta T \ll \epsilon_0$, where $a$ is the coefficient of thermal expansion of the body and $\Delta T$ is the temperature increment at local melting. These simplifications allow for the direct application of the theory of elastic-plastic deformation.

Note that all the considerations and results of this paper do not depend on whether the spherical region contains the fluid phase of the matter or an isotropic solid phase. It is only essential that the specific volume of the new phase exceeds the specific volume of the preceding phase.

**1. Deformation of the solid body during melting (loading).** Consider an infinite isotropic body with a spherical cavity of radius $r_0$ filled with a fluid. The radius of this cavity increases in time; it is assumed, however, that $r_0$ is much smaller than the velocity of sound in this medium. The latter assumption allows for assuming the system fluid – solid body as being in equilibrium for every fixed $r_0$.

Spherical symmetry of the problem implies that some components of stress and strain tensors are identical

$$\sigma_{\varphi\varphi} = \sigma_{\psi\psi} \equiv \sigma_{\varphi}, \qquad u_{\varphi\varphi} = u_{\psi\psi} \equiv \varepsilon_{\varphi}$$

($r$, $\phi$, $\psi$ are spherical coordinates whose origin is in the center of the fluid sphere) and the off-diagonal components of these tensors vanish.*

* The non-vanishing components of the tensor $u_{ik}$ may be expressed in

Thus, the intensity of shear stresses* is proportional to $\sigma_\phi - \sigma_r$ ($\sigma_r \equiv \sigma_{rr}$), the intensity of shear strains is proportional to $\epsilon_\phi - \epsilon_r$ ($\epsilon_r \equiv u_{rr}$) and each element of the solid body is subjected to proportional loading [1].

The components of the stress tensor $\sigma_{ik}$ are related by the equations of equilibrium, and the components $u_{ik}$ are related by the compatibility equations. In the fluid, the pressure $p$ is constant over its volume if gravity forces are neglected. The tensors $\sigma_{ik}$ and $u_{ik}$ satisfy the following boundary conditions:

1) The surface of the body at infinity is free of stresses

$$\sigma_r = 0 \qquad \text{for} \quad r = \infty \tag{1.1}$$

2) On the boundary of the solid body and the fluid, the surface stresses are continuous

$$\sigma_r = -p \quad \text{for} \quad r = r_0 \tag{1.2}$$

and the deformations are continuous

$$\varepsilon_\varphi = \varepsilon_0 - k_1 p \qquad \text{for} \quad r = r_0 \tag{1.3}$$

where $k_1$ is the coefficient of compressibility of the fluid, and $3\epsilon_0$ is the increment of specific volume during melting.

In order to determine $\sigma_{ik}$ and $u_{ik}$ which satisfy the conditions (1.1) to (1.3), it is necessary to establish the relation between the tensors $\sigma_{ik}$ and $u_{ik}$. Using the relation between the volume deformation and the mean pressure, which is valid in a wide range of volumetric deformation, the following relations are easily obtained:

---

terms of the only non-vanishing radial component $u_r$ of the displacement vector:

$$u_{\varphi\varphi} = r^{-1}u_r, \qquad u_{rr} = du_r / dr.$$

* The intensity of shear stresses $I_2(\sigma)$ is defined in the usual way as

$$I_2^2(\sigma) = \frac{1}{2}(\sigma_{ik} - \frac{1}{3}\delta_{ik}\sigma_{ll})^2$$

(repeated indices denote summation). The intensity of shear strains is similarly expressed in terms of the strain tensor.

$$\varepsilon_\varphi - \varepsilon_r + 2k(\sigma_\varphi - \sigma_r) = \frac{C}{r^3} \tag{1.4}$$

$$\varepsilon_\varphi - k\sigma_r = \frac{C}{3r^3}, \qquad C = \text{const} \tag{1.5}$$

where $k$ is the coefficient of compressibility of the solid body.

Equations (1.5) and (1.3) give the relation between $C$ and the pressure $p$ in the fluid

$$\frac{C}{3r_0^3} = \varepsilon_0 + (k - k_1)\,p \tag{1.6}$$

The pressure in turn may be expressed in terms of the stress intensity

$$p = 2\int_{r_0}^{\infty} \frac{\sigma_\varphi - \sigma_r}{r}\,dr \tag{1.7}$$

if relations (1.1) and (1.2) are used.

It is essential that relations (1.4) and (1.5) (and also (1.6) to (1.8) derived from them) do not depend on the type of relation between the intensity of shear stresses and the intensity of shear strains.

Equations (1.6) and (1.7) may be transformed into one equation for the determination of the constant $C$:

$$\frac{C}{3r_0^3} = \varepsilon_0 + 2(k - k_1)\int_{r_0}^{\infty} \frac{\sigma_\varphi - \sigma_r}{r}\,dr \tag{1.8}$$

In order to assign a meaning to Equation (1.8), it is necessary to find the variation of $\sigma_\phi - \sigma_r$ in $r$, which is determined by the relation between $\epsilon_\phi - \epsilon_r$ and $\sigma_\phi - \sigma_r$. The theory of elastic-plastic deformations [1] establishes this relation for the increasing loading in the form

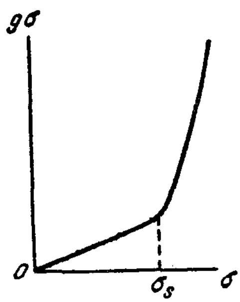

Fig. 1.

$$\varepsilon_\varphi - \varepsilon_r = g(\sigma)\,\sigma, \qquad \sigma = \sigma_\varphi - \sigma_r \tag{1.9}$$

where $g(\sigma)$ is a monotonically increasing even function which is constant for small $\sigma$; $g(\sigma) \to 2\mu$ for $\sigma \to 0$. The diagram of the function $g(\sigma)\,\sigma$ is schematically represented in Fig. 1.

If (1.9) is substituted into (1.4), the relation $\sigma = \sigma(r)$ is determined in the implicit form by the equation

$$G(\sigma) = \frac{C}{r^3}, \qquad G(\sigma) = [2k + g(\sigma)]\,\sigma \tag{1.10}$$

The constant $C$ can be found from (1.8), as previously indicated.

In the following, it will be advantageous to introduce, instead of $C$, another constant $\sigma_0 = \sigma(r_0)$, which is related to $C$ by the equation

$$C = r_0^3 G(\sigma_0)$$

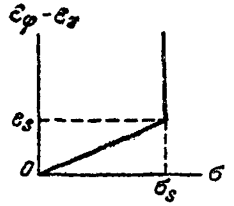


Fig. 2.

The equation for $\sigma_0$ follows from (1.8) and (1.10).

The introduction of the constant $\sigma_0$ allows the writing of the expression for the fluid pressure in the form

$$p = \frac{2}{3} \int_0^{\sigma_0} \frac{G'(\sigma)}{G(\sigma)} \sigma d\sigma \qquad (1.11)$$

Equation (1.11) indicates that the fluid pressure does not depend on $r_0$ (which is a consequence of the dimensional properties of this problem), and is determined by the quantity $\epsilon_0$. For $\epsilon_0 = 0$ the fluid pressure becomes zero.

If the compressibilities of the fluid and the solid body are identical ($k = k_1$), then the stress $\sigma_0$ is determined by the equation

$$G(\sigma_0) = 3\varepsilon_0 \qquad (1.12)$$

In the case of Prandtl's model (Fig. 2), which assumes that $\epsilon_\phi - \epsilon_r$ and $\sigma = \sigma_\phi - \sigma_r$ are related by different functional relations below and above the yield point ($e_s$, $\sigma_s$)

$$\begin{aligned} \varepsilon_\varphi - \varepsilon_r &= 2\mu\sigma && \text{for } \sigma < \sigma_s \\ \sigma = \sigma_s &= \text{const} && \text{for } \varepsilon_\varphi - \varepsilon_r > e_s = 2\mu\sigma_s \end{aligned} \qquad (1.13)$$

the pressure $p$ is given by the expression

$$p = \frac{2}{3}\sigma_s \left[1 + \ln\left(\frac{a}{r_0}\right)^3\right] \qquad (1.14)$$

where $a$ is the radius of the plastic zone [1 ], determined by the equation

$$\left(\frac{a}{r_0}\right)^3 + \frac{1}{3}\frac{k_1 - k}{k + \mu}\left[1 + \ln\left(\frac{a}{r_0}\right)^3\right] = \frac{\varepsilon_0}{2(k + \mu)\sigma_s} \qquad (1.15)$$

From (1.15), for $k = k_1$, follows the relation

$$\left(\frac{a}{r_0}\right)^3 = \frac{\varepsilon_0}{2\sigma_s(k + \mu)}$$

and, for more general cases, the order of magnitude may be estimated by

$$\left(\frac{a}{r_0}\right)^3 \sim \frac{\varepsilon_0}{e_s}$$

**2. Deformation of the solid body during hardening (unloading).** After the process of melting is terminated, and the radius of the fluid reaches its maximum value $r_m$, the reverse process of hardening of the fluid starts. The hardening of the fluid phase on the boundary $r = r_0$ is equivalent to the formation of a thin layer of the solid phase on the deformed and stressed base. Here, two cases are possible.

*Case A.* The growing solid phase carries elastic stresses developed in the base. It is assumed, thus, that in the newly created, infinitely thin layer cf the solid body an instantaneous stress arises which is described by the equation

$$\varepsilon_\varphi - \varepsilon_r = g_0(\sigma)\,\sigma \tag{2.1}$$

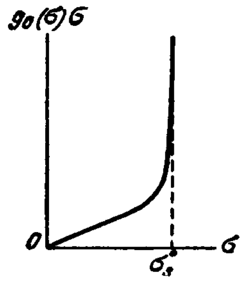

Fig. 3.

where $g_0(\sigma)$ is a monotonically increasing function, in general different from $g(\sigma)$, and having the following properties:

1. For small $\sigma$, the function $g_0(\sigma)$ has a horizontal section ($g_0(\sigma) \to 2\mu$ for $\sigma \to 0$);

2. As $\sigma$ approaches $\sigma_0^\circ$, the function $g_0(\sigma)\,\sigma$ approaches asymptotically the vertical line $\sigma = \sigma_s^\circ$.

In the course of hardening ($\dot{r}_0 < 0$), the motion of the boundary causes the unloading of the material around the melted region. As usual, the unloading is described by a linear relation between the changes of strain and stress tensors

$$d(\varepsilon_\varphi - \varepsilon_r) = 2\mu\, d\sigma \tag{2.2}$$

Using the differential equation (2.2) for the relation between finite changes, it is necessary to remember that the origins of unloading are different in the regions $r > r_m$ and $r < r_m$. In particular

$$\varepsilon_\varphi - \varepsilon_r = 2\mu\sigma + [g(v) - 2\mu]\,v \qquad (r > r_m) \tag{2.3}$$

$$\varepsilon_\varphi - \varepsilon_r = 2\mu\sigma + [g_0(\zeta) - 2\mu]\,\zeta \qquad (r < r_m) \tag{2.4}$$

where $v$ is the intensity of stress $\sigma$ for $r_0 = r_m$, and it is determined by Equations (1.10) and (1.12) with $r_0 = r_m$; $\zeta$ is the value of $\sigma$ in the region of solidified material at the start of unloading, and corresponds to maximum loading at a given point. The quantity $\zeta$ is determined by the intensity of stresses at a given point at the time when the boundary passes it:

$$\sigma = \zeta \qquad \text{for} \quad r = r_0 \tag{2.5}$$

With the use of (1.4), relations (2.3) and (2.4) yield the expressions for $\sigma$ analogous to (1.10):

$$2(\mu + k)\sigma + [g(v) - 2\mu]\, v = \frac{C}{r^3} \qquad (r > r_m) \tag{2.6}$$

$$2(\mu + k)\sigma + [g_0(\zeta) - 2\mu]\, \zeta = \frac{C}{r^3} \qquad (r < r_m) \tag{2.7}$$

Because of the continuity of $\sigma_r$ and $\epsilon_\phi$ at the point $r = r_m$, the constant $C$ is the same in both (2.6) and (2.7). This constant is related to the value of the function $\zeta = \zeta(r)$ for $r = r_0$. Indeed, (2.5) and (2.6) imply

$$C = r_0^3 G_0(\zeta), \qquad G_0(\zeta) = [2k + g_0(\zeta)]\, \zeta, \qquad \zeta_0 = \zeta(r_0) \tag{2.8}$$

Determining the stress intensity from (2.6) and (2.7), substituting it into (1.8) and using (2.8), the equation for the function $\zeta = \zeta(r)$ is obtained:

$$G_0(\zeta_0) = 3\varepsilon_0 + 6(k - k_1)\int_{r_0}^{\infty} \frac{\sigma}{r}\, dr \tag{2.9}$$

The integral equation (2.9) determined $\zeta(r)$. Since the function $v$ does not depend on $r_0$, differentiation of (2.9) with respect to $r_0$ gives

$$r_0 \frac{d\zeta_0}{dr_0} = \beta \frac{g_0(\zeta_0) - 2\mu}{G_0'(\zeta_0)} \zeta_0, \qquad \beta = 3\frac{k - k_1}{k_1 + \mu} \tag{2.10}$$

Integration of (2.10) gives

$$\beta \ln \frac{r}{r_m} = \int_{\zeta_m}^{\zeta} \frac{G_0'(\zeta)\, d\zeta}{[g_0(\zeta) - 2\mu]\zeta} \tag{2.11}$$

The relation (2.11), together with (2.6) and (2.7), determines the distribution of stresses in the solid body during the process of unloading.

From the known distribution of the stress intensity in the solid body, the fluid pressure $p = p(r_0)$ may be easily determined; during unloading it depends on the position of the boundary, i.e. on $r_0$. The pressure is determined by Equation (1.7) using (2.6) to (2.9):

$$p = p_m - \frac{G_0(\zeta_m) - G_0(\zeta_0)}{3(k-k_1)} \equiv \frac{G_0(\zeta_0) - 3\varepsilon_0}{3(k-k_1)} \qquad (p_m = p(r_m)) \tag{2.12}$$

Here, $p_m$ denotes the pressure at the beginning of hardening (the pressure produced by melting).

The dependence of the pressure on $\dot{r}_0$ becomes especially simple if $k = k_1$. Then, from (2.10) follows $\zeta \equiv \zeta_0{}^* = \text{const}$, and the pressure may be conveniently determined directly from relation (1.7):

$$p = p_m + \frac{g_0(\zeta^*) - 2\mu}{k + \mu} \zeta^* \ln\left(\frac{r_0}{r_m}\right) \tag{2.13}$$

The pressure in the fluid decreases during unloading with progressive hardening ($\dot{r}_0 < 0$). In fact, the properties of the function $g_0(\zeta_0)$ imply that for $\zeta_0 > 0$

$$\frac{dp}{dr_0} = \frac{g_0(\zeta_0) - 2\mu}{(\mu + k_1)r_0} \zeta_0 \geqslant 0$$

where the equality $dp/dr_0 = 0$ occurs only for the elastic type of relation (2.5), i.e. for $g_0(\zeta) = 2\mu$. Because during hardening $\dot{r}_0 < 0$, it must be $\dot{p} < 0$.

It is easy to see that the decreasing $p(r_0)$ assumes value zero, i.e. the value of $r_0 > 0$ exists (denoted below by $\rho_0$) for which $p = 0$. This value of $r_0$ satisfies the condition following from (2.12)

$$3\varepsilon_0 = G_0(\zeta_0) \tag{2.14}$$

Relation (2.14) may be considered as the equation for $\zeta_0$. If the root of this equation is denoted by $\zeta^*$ (it is significant that this quantity has entered into (2.13)), then $\rho_0$ may be determined explicitly by relation (2.11):

$$\beta \ln \frac{\rho_0}{r_m} = \int_{\zeta_m}^{\zeta^*} \frac{G_0'(\zeta)\, d\zeta}{[g_0(\zeta) - 2\mu]\, \zeta} \tag{2.15}$$

For $r_0 < \rho_0$ the fluid pressure is negative.

If the compressibilities of the fluid and the solid body are identical ($k = k_1$), then from Equation (2.13)

$$\ln \frac{\rho_0}{r_m} = -\frac{(k + \mu)\, p_m}{[g_0(\zeta^*) - 2\mu]\, \zeta^*}$$

*Case B*. The growing solid phase deforms in such a way that it undergoes only uniform pressure. In this case, shearing stresses in a newly-created, infinitely thin layer of the solid phase are assumed to be equal to zero*:

$$\sigma = 0 \qquad \text{for } r = r_0 \tag{2.16}$$

Therefore, instead of (2.4) the relation between $\epsilon_\phi - \epsilon_r$ and $\sigma$ is

$$\varepsilon_\varphi - \varepsilon_r = 2\mu\sigma + q_0 \qquad (r < r_m) \tag{2.17}$$

where $q_0$ is the value of the strain intensity $\epsilon_\phi - \epsilon_r$ in the region of hardening at the beginning of unloading. The quantity $q_0$ is determined by the intensity of strain at a given point at the time when the boundary surface passes this point.

Indeed, from (2.16) and (2.17)

$$\varepsilon_\varphi - \varepsilon_r = q_0 \qquad \text{for } r = r_0$$

Here, the function $q_0 = q_0(r)$ is to be determined.

Comparison of (2.16) and (2.5) with (2.17) and (2.4) indicates that the results obtained for Case A may be used for the analysis of stresses and strains if $\zeta \equiv 0$ and $g_0(\zeta)\zeta \equiv q_0$ are assumed. (It is only necessary to take into account that $q_0(r)$ is an unknown function of $r$, while $g_0(\zeta)$ was assumed as a known function of the unknown quantity $\zeta$). The equation for $q_0(r)$ follows directly from (2.10):

$$\frac{dq_0}{q_0} = \beta \frac{dr}{r} \tag{2.18}$$

and the boundary condition for $r = r_m$ follows from (2.11):

$$q_0(r_m) = 3\varepsilon_0 + 6(k - k_1) \int_{r_m}^{\infty} \frac{\sigma}{r} dr \tag{2.19}$$

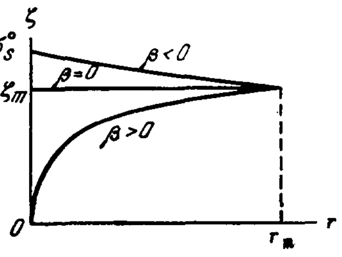


Fig. 4.

where the value of $\sigma$ is, as previously, determined from the condition (1.10). From (2.17) and (2.18), a simple expression for $q_0(r)$ follows:

$$q_0(r) = [3\varepsilon_0 + 3(k - k_1)p_m]\left(\frac{r}{r_m}\right)^\beta \tag{2.20}$$

In writing (2.20), Expression (1.7) was used. The fluid pressure is

---

* Formally, Case B corresponds to $\sigma_s^\circ \equiv 0$ in Case A.

obtained from (2.12):

$$p = \frac{q_0(r_0) - 3\varepsilon_0}{3(k - k_1)}$$

As in Case A, the pressure becomes zero at the point $\rho_0$ determined by the equation $q_0(\rho_0) = 3\epsilon_0$, with $\partial p/\partial r_0 > 0$ at this point. Consequently, as in Case A, the fluid pressure becomes negative for $r_0 < \rho_0$.

**3. Analysis of the state of stress in the body during unloading.** For a qualitative investigation of the state of stress of the solid body and the dependence of the pressure of the fluid on $r_0$, a simple model of plasticity of solids is considered, i.e. Prandtl's model. This means, in particular, that the function $g_0(\sigma)\,\sigma$, introduced in the preceding section, is determined analogously to (1.13), and its diagram has the form given in Fig. 2 but with a different yield point $(\sigma_s^\circ,\ e_s^\circ)$.

Specific applications of the presented theory are carried out in different ways for different signs of $\beta$. This results from the fact that the function $\zeta$, whose value determines the intensity of stresses at the beginning of unloading, behaves differently, depending on the sign of $\beta$.

For $\beta > 0$, as follows from (2.10), the quantity $\zeta$ is an increasing function of $r$, becoming zero for $r = 0$, and reaching $\zeta_m$ for $r = r_m$ (Fig. 4). If $\beta < 0$, then $\zeta$ is a decreasing function of $r$, which, on the basis of the properties of the function $g_0(\sigma)\,\sigma$ assumed in Section 2, is equal to $\sigma_s^\circ$ $(\sigma_s^\circ > \zeta_m)$ for $r = 0$ and equal to $\zeta_m$ for $r = r_m$ (Fig. 4).

For $\beta = 0$, the function $\zeta$ reduces to a constant $\zeta = \zeta_m$.

In the proposed theory, the part of $\zeta_m$ is played by the quantity $\sigma_s^\circ$ from which the unloading starts at the beginning of hardening. This theory does not admit for newly-hardened layers the values of $\sigma$ larger than $\sigma_0^\circ$. Therefore, if $\beta \leqslant 0$, the beginning of unloading in an arbitrary point $r$ coincides with the stress intensity $\sigma_s^\circ$; if $\beta > 0$, a region of small $r$ necessarily exists in which instantaneous loading in newly-created layers of solid body does not exceed the "yield point", and $\sigma < \sigma_s^\circ$ corresponds to the beginning of unloading.

Because the coefficient of compressibility of the fluid phase is larger than the corresponding coefficient of the solid phase of a given material $(k_1 > k)$, the discussion may be limited to the case $\beta < 0$.

In this case, instead of (2.4) the relation

$$\varepsilon_\varphi - \varepsilon_r = 2\mu\,(\sigma - \sigma_s^\circ) + q_s(r), \qquad r < r_m$$

should be used, which coincides with (2.17) for $\sigma_s^\circ \equiv 0$. The quantity $q_s(r)$, as in (2.17), is determined by stress intensity at the point $r$ at the moment when the boundary surface passes this point. In order to find $q_s(r)$, it is again convenient to use directly the results of Case A, Section 2, noting that the formal replacement in all expressions $\zeta \to \zeta_s^\circ$ and $g_0(\zeta)\zeta \to q_s(r)$ is sufficient. Thus, the expression for $q_s(r)$ is obtained:

$$\frac{dq_s}{q_s - 2\mu\sigma_s^\circ} = \beta\frac{dr_0}{r_0} \tag{3.1}$$

with the boundary condition

$$q_s(r_m) + 2k\sigma_s^\circ = 3\varepsilon_0 + 3(k - k_1)p_m \tag{3.2}$$

From (3.1) and (3.2) follows

$$q_s(r) - 2\mu\sigma_s^\circ = 3\left(\frac{r}{r_m}\right)^\beta [\varepsilon_0 + (k - k_1)p_m - \tfrac{2}{3}(\mu + k)\sigma_s^\circ] \tag{3.3}$$

Note that the characteristics of the process of unloading in the region $r > r_m$ do not enter explicitly into (3.3), but that only $p_m$ appears. Nevertheless, the form of the function $q_s(r)$, as well as Equation (3.1), is determined by the linear character of unloading. Using (3.3) and (2.14), the equation

$$p = p_m - \frac{(r_0/r_m)^\beta - 1}{k_1 - k}[\varepsilon_0 + (k - k_1)p_m - \tfrac{2}{3}\sigma_s^\circ(\mu + k)] \tag{3.4}$$

may be obtained.

The pressure becomes zero at the point $r_0 = \rho_0$, which may be found either from (3.4) or directly from (2.15):

$$\left(\frac{\rho_0}{r_m}\right)^{-\beta} = 1 - \frac{(k_1 - k)p_m}{\varepsilon_0 - \frac{2}{3}(k + \mu)\sigma_s^\circ} \tag{3.5}$$

The case of $\epsilon_0 \gg e_s$ is most interesting, because for high temperatures the yield point is low and the change of volume during melting is large (usually $\epsilon_0 \sim 10^{-2}$). In this case, the point $r_0 = \rho_0$ is very close to $r_0 = r_m$. In fact, because $(n(a/r_0)^3$ in relation (1.14) depends only slightly on the ratio $\epsilon_0/e_s$, it is approximately $p_m \sim \sigma_s$, and, consequently, the second component of the right-hand side of (3.5) may be estimated as

$$\frac{(k - k_1)p_m}{\varepsilon_0 - {}^2/_3(k + \mu)\sigma_s^\circ} \sim \frac{e_s}{\varepsilon_0} \ll 1$$

This estimate permits us to set

$$\rho_0 / r_m = 1 - \eta \qquad (\eta \ll 1)$$

In the linear approximation with respect to $\eta$, Equation (3.5) gives

$$\eta = \frac{(k_1 + \mu)\, p_m}{3\varepsilon_0 - 2(k + \mu)\, \sigma_s^\circ} \approx \frac{(k_1 + \mu)\, p_m}{3\varepsilon_0} \tag{3.6}$$

As might be expected, $\eta$ is not too sensitive to $\sigma_s^\circ$.

If $r_0 < \rho_0$, the pressure in the fluid becomes negative, and its absolute value rapidly increases as $r_0$ decreases. It is easy to verify that if the boundary surface moves from the point $r_0 = r_m$ at the distance $2\eta$, the absolute value of pressure reaches

$$p = -p_m \qquad \text{for} \;\; r_0 = r_m - 2\eta \tag{3.7}$$

In this way, even for a relatively small amount of "freezing" of the fluid, the negative pressure becomes very large (in its absolute value).

Because of $\beta < 0$, for very small $r_0/r_m$, the negative pressure determined by Expression (3.4) should increase in its absolute value as $(r_m/r_0)^{-\beta}$. But the stress intensity at some points would exceed (in its absolute value) the limit value $\sigma_s$, i.e. the material at these points would be beyond the elastic limit. This means that for small $r_0$ linear unloading in some regions becomes plastic loading of opposite sign. The analysis of the stress intensities, which are found from (2.6) and (2.7) using (3.3), actually shows that for small $r_0/r_m$ the absolute value of $\sigma$ becomes very large, and it may reach $\sigma_s$ in the vicinity of $r = r_m$.

In the derivation of the relations discussed in this section, the essential assumption has been made that the unloading has linear character, and, consequently, it is necessary to stay within the limits for which this assumption is valid. Therefore, if $\epsilon_0 \gg e_s$, all the derived relations may be used only for small $(r_m - r_0)/r_m$, namely for

$$\frac{r_m - r_0}{r_m} < \frac{e_s}{\varepsilon_0}$$

Nevertheless, for small $(r_m - r_0)/r_m$ also, as, for instance, (3.7) indicates, the negative pressure in the fluid may be very large.

But it is known [2] that the state with a large negative pressure is a non-equilibrium state and its stability is limited. In other words, in such a state the fluid tends to compress itself by separating from the solid walls or by creating internal cavities. This tendency is counteracted only by the fact that during fractures new surfaces are created which increase the surface energy of the system. If the increment of the surface energy is compensated by the decrement of the energy of volumetric deformation, the fluid will necessarily become discontinuous.

If in the fluid of the volume $\sim r_{ж}^3$ a discontinuity occurs, then the fluid pressure becomes zero with the velocity of sound, and further hardening continues at $p = 0$. As a result of this, cavities of the volume $\sim \epsilon_0 r_{ж}^3$ are created in the solid body by changes of the specific volume of the material after full crystallization of the fluid.

The authors consider it their duty to express gratitude to I.M. Lifshitz for valuable discussions.

BIBLIOGRAPHY

1. Kachanov, L.M., *Osnovy teorii plastichnosti (Foundations of the Theory of Plasticity)*. GITTL, 1958.

2. Landau, L.D. and Lifshitz, E.M., *Statisticheskaia fizika (Statistical Physics)*. GITTL, 1951.

*Translated by* M.P.B.